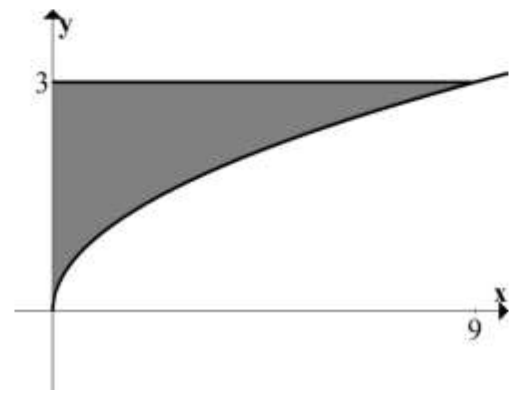
Even more practice describing regions

For each of the practice examples, show how you would set up the bounds for a double integral over the given region in both ways: bottom/top and left/right.

$$\iint_D f(x,y)dA = \int_a^b \left(\int_{BOTTOM(x)}^{TOP(x)} f(x,y)\,dy \right) dx = \int_c^d \left(\int_{LEFT(y)}^{RIGHT(y)} f(x,y)\,dx \right) dy$$

Example 1: Consider the region bounded by the curves $y = \sqrt{x}, x = 0, y = 3$.



Top/Bottom Answer:

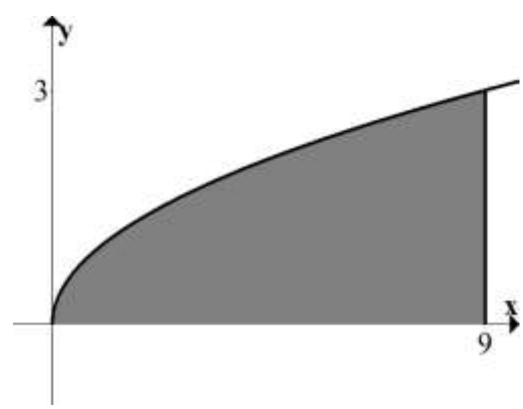
For any fixed *x* chosen from $0 \le x \le 9$, we see $\sqrt{x} \le y \le 3$.

Left/Right Answer:

For any fixed *y* chosen from $0 \le y \le 3$, we see $0 \le x \le y^2$.

$$\int_0^9 \left(\int_{\sqrt{x}}^3 f(x,y)\,dy \right) dx \qquad OR \qquad \int_0^3 \left(\int_0^{y^2} f(x,y)\,dx \right) dy$$

Example 2: Consider the region bounded by the curves $y = \sqrt{x}, x = 9, y = 0$.



Top/Bottom Answer:

For any fixed *x* chosen from $0 \le x \le 9$, we see $0 \le y \le \sqrt{x}$.

Left/Right Answer:

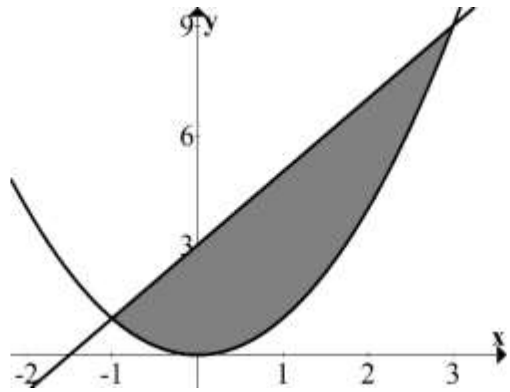
For any fixed *y* chosen from $0 \le y \le 3$, we see $y^2 \le x \le 9$.

$$\int_0^9 \left(\int_0^{\sqrt{x}} f(x,y)\,dy \right) dx \qquad OR \qquad \int_0^3 \left(\int_{y^2}^9 f(x,y)\,dx \right) dy$$

Example 3: Consider the region bounded by the curves $y = x^2, y = 2x + 3$.
Note that the graphs intersect at (-1,1) and (3,9).

Top/Bottom Answer:

For any fixed *x* chosen from $-1 \le x \le 3$, we see $x^2 \le y \le 2x + 3$.



Left/Right Answer: Note that this is a poor choice since the equation on the left changes at y = 1. To do this, we would have to break up the problem into two regions as follows:
For any fixed *y* chosen from $0 \le y \le 1$, we see $-\sqrt{y} \le x \le \sqrt{y}$.

For any fixed *y* chosen from $1 \le y \le 9$, we see $\frac{y-3}{2} \le x \le \sqrt{y}$.

$$\int_{-1}^{3}\left(\int_{x^2}^{2x+3} f(x,y)\, dy\right) dx \qquad OR \qquad \int_{0}^{1}\left(\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\, dx\right) dy + \int_{1}^{9}\left(\int_{(y-3)/2}^{\sqrt{y}} f(x,y)\, dx\right) dy$$

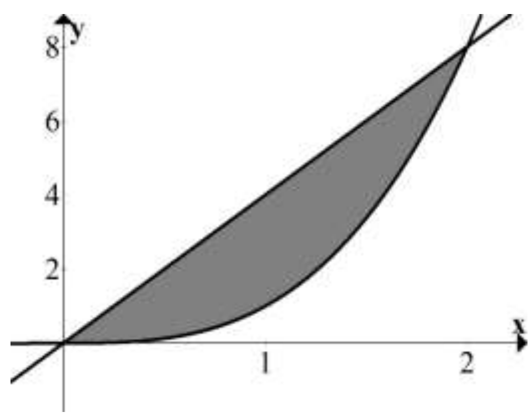
Example 4: Consider the region bounded by the curves $y = x^3, y = 4x$.

Top/Bottom Answer:

For any fixed *x* chosen from $0 \le x \le 2$, we see $x^3 \le y \le 4x$.

Left/Right Answer:

For any fixed *y* chosen from $0 \le y \le 8$, we see $y/4 \le x \le y^{\frac{1}{3}}$.



$$\int_{0}^{2}\left(\int_{x^3}^{4x} f(x,y)\, dy\right) dx \qquad OR \qquad \int_{0}^{8}\left(\int_{y/4}^{y^{\frac{1}{3}}} f(x,y)\, dx\right) dy$$

For the next two examples, **draw the region that goes with the given double integral, then <u>reverse the order of integration</u>.**

Example 5: $\displaystyle\int_0^3 \left(\int_{x^2}^{3x} f(x,y)\, dy \right) dx$

Answer: We are given a TOP/BOTTOM description!!!

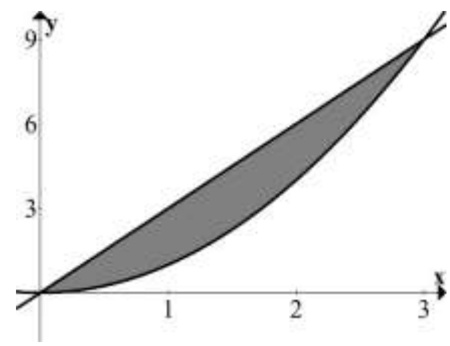
Start by drawing $y = x^2$ (will be the BOTTOM or lower bound!)

and $y = 3x$ (will be the TOP or upper bound!)

Now reverse the order

For $0 \le y \le 9$, we see $\frac{y}{3} \le x \le \sqrt{y}$. Thus,

$$\int_0^3 \left(\int_{x^2}^{3x} f(x,y)\, dy \right) dx = \int_0^9 \left(\int_{y/3}^{\sqrt{y}} f(x,y)\, dx \right) dy$$



Example 6: $\displaystyle\int_0^4 \left(\int_6^{2y+6} f(x,y)\, dx \right) dy$

Answer: We are given a LEFT/RIGHT description!!!

Start by drawing $x = 6$ (will be the LEFT or lower bound!)

and $x = 2y + 6$ (will be the RIGHT or upper bound!)

Now reverse the order

For $6 \le y \le 14$, we see $\frac{x-6}{2} \le y \le 4$. Thus,

$$\int_0^4 \left(\int_6^{2y+6} f(x,y)\, dx \right) dy = \int_6^{14} \left(\int_{(x-6)/2}^{4} f(x,y)\, dy \right) dx$$